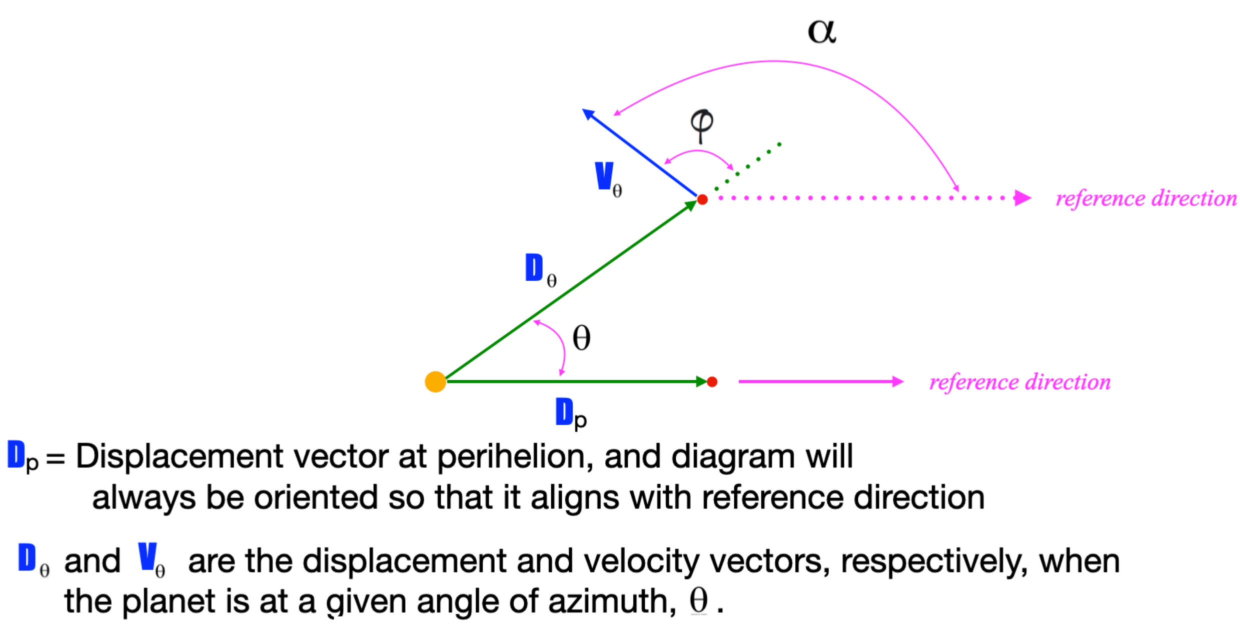
Synopsis of "Feynman's Lost Lecture and Beyond"

<u>Part 1: Introduction and Historical Context</u>

This video is relatively short and self-explanatory.

<u>Part 2: Definitions</u>

This video reviews some of the basic geometry of ellipses, and then provides definitions of terms and variables used in subsequent video presentations. These definitions are summarized in the following diagram:



Note: In this synopsis, a vector will be represented in bold blue (eg, $\mathbf{V}$), and a vector's magnitude will be represented in script light blue (eg, $\mathcal{V}$). In the video presentations, a more standard form of vector notation is used.

<u>Part 3: Proof of Kepler's Second Law</u>

This video offers my representation of Feynman's representation of Newton's proof of Kepler's second law. Using a series of triangles to approximate the area swept out by a planet as it orbits the sun, and then shrinking the central angle of each triangle down towards zero, one can readily show that, in modern notation,

$$\frac{\Delta A}{\Delta t} = h/2\text{, a constant}$$

where h is the specific angular momentum, L/m, such that
h = $\mathcal{D}$ x $\mathcal{V}$ x sin φ

Furthermore, , as $\Delta t \longrightarrow 0$,

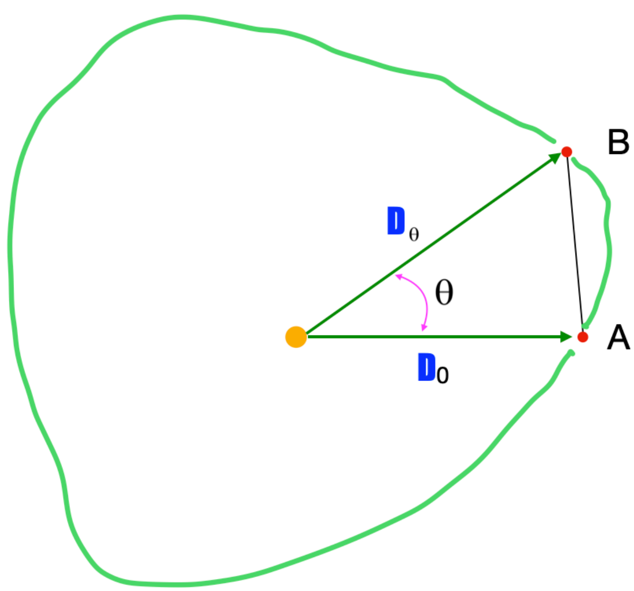
$$\frac{\Delta A}{\Delta t} = dA/dt = h/2$$

In this manner, Newton's introduction of an intuitive concept of limits and infinitesimals leads naturally to the concept of a derivative. Because the derivative here is a constant and is easy to visualize geometrically, it sneaks in unannounced so to speak, without the need for any formal definition, just as the conservation of angular momentum, for a planet orbiting the sun, sneaks in unannounced.

<u>Part 4: The Special Case of Circular Orbits</u>
This video examines the simple case of a hypothetical circular orbit, the consideration of which leads to some key insights that will be very helpful when exploring the case of non-circular orbits.

<u>Parts 5.1 to 5.3: An Ellipse? *Etc*</u>
These videos comprise my representation of Feynman's development of what I call the d**V**/d-theta circle. Because the d**V**/d-theta circle is so essential to this novel approach to the central force problem, I provide a fairly detailed synopsis here.

Consider a planet traveling from point A to point B along its arbitrary orbital path:



The area swept out is approximately

$1/2 \times \mathcal{D}_0 \times \mathcal{D}_\theta \times \sin\theta$

As we shrink θ down so that $\theta \longrightarrow 0$

$\mathcal{D}_\theta \rightarrow \mathcal{D}_0$ and $\sin\theta \rightarrow \theta \rightarrow d\theta$

So, $$\frac{dA}{d\theta} = 1/2 \times \mathcal{D}^2$$

Now, taking into account Newton's law of gravitation, which we can write as
F/m = acceleration = $d\mathcal{V}/dt = GM/D^2$, we find that

$$\frac{dA}{d\theta} \times \frac{d\mathcal{V}}{dt} = GM/2$$

Rearranging and substituting, we get

$$\frac{dA}{dt} \times \frac{d\mathcal{V}}{d\theta} = GM/2$$

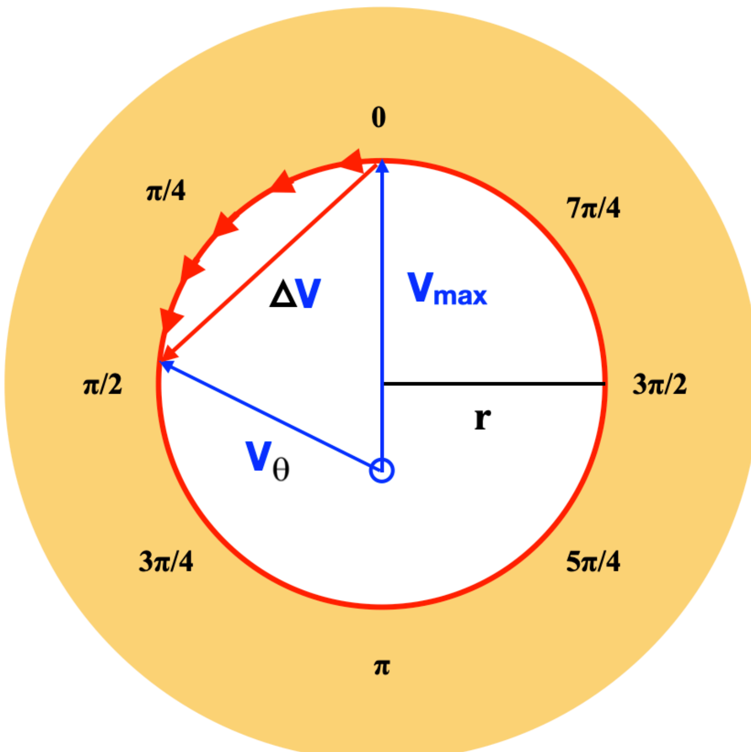
$$\frac{h}{2} \times \frac{d\mathcal{V}}{d\theta} = GM/2$$

$$\frac{d\mathcal{V}}{d\theta} = GM/h, \text{ a constant}$$

The fact that $d\mathcal{V}$/d-theta is constant is significant. If we line up "all of the $d\mathbf{V}$'s" in order, tail to head, for the entire orbit, in vector space, maintaining their same orientation relative to the same reference direction as in position space, they form a circle (ie, a regular polygon with an infinite number of sides)—the $d\mathbf{V}$/d-theta circle.The radius of the $d\mathbf{V}$/d-theta circle is GM/h, which has units of velocity.

For a circular orbit, $\mathcal{V}$ = GM/h. For a non-circular orbit, it can be shown that the maximum velocity, $\mathbf{V}_{max}$ = (1 + e)GM/h, where e, as I prove in a subsequent video, is the eccentricity of the orbit and, as one would expect, has a magnitude such that $1 < e < 2$. Moreover, proper placement of $\mathbf{V}_{max}$ in the $d\mathbf{V}$/d-theta circle of vector space requires that it lie along the vertical diameter of the circle with its head touching the top of the circle. To find $\mathbf{V}_{theta}$ associated with a planet at any angle of azimuth, one draws a vector with its tail on the tail of $\mathbf{V}_{max}$ and its head on the circle at an arc length of theta radians from the head of $\mathbf{V}_{max}$.

In the diagram below, numbers in the yellow circle represent angles of azimuth. The head of any given velocity vector will touch the $d\mathbf{V}/d\theta$ circle at a point such that the arc length in radians, counter-clockwise from 0 (ie, the top of the circle) to that point gives the angle of azimuth of the planet when it has that velocity.



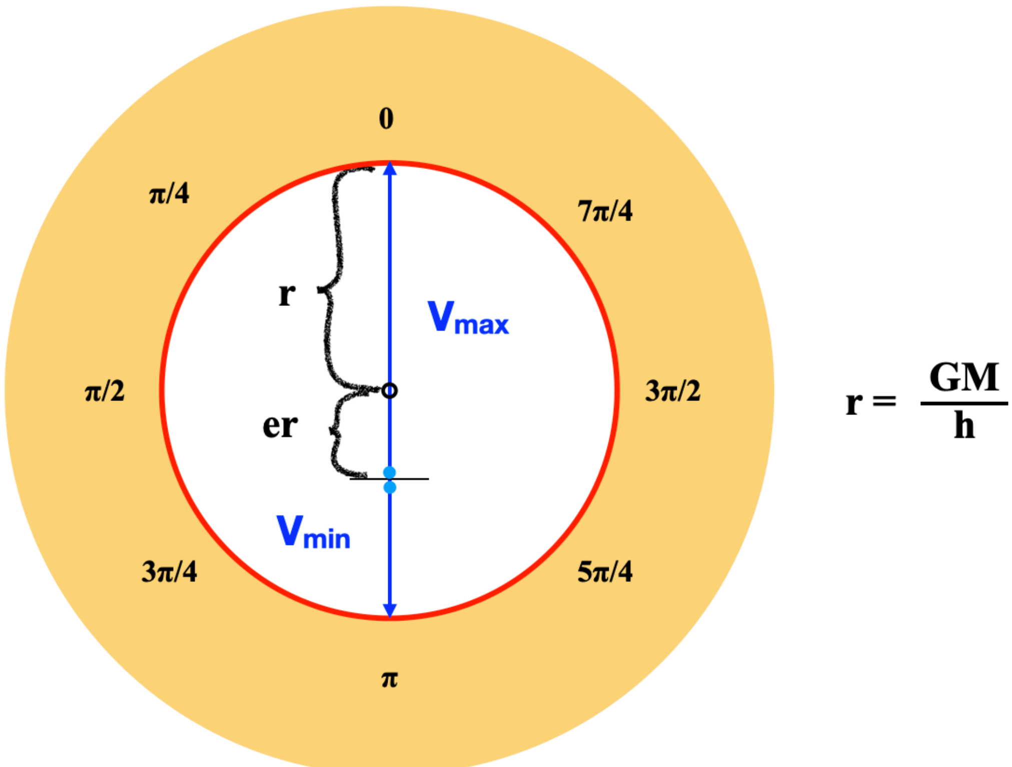
$$r = \frac{GM}{h}$$

$d\mathbf{V} / d\theta$ **circle**

Thus, the $d\mathbf{V}$/d-theta circle provides a simple means of "integrating" $d\mathbf{V}$'s from 0 to theta, such that their sum produces the resultant vector delta $\mathbf{V}$. Once again, a little bit of calculus sneaks in unannounced because of the visually intuitive nature of the integral. If everything were drawn to scale, one could directly measure the magnitude and the angle, alpha, of the vector. Feynman instead goes on to construct his geometric proof of the law of ellipses, and this is where I leave him to go in a different direction.

<u>Part 5.4 and 5.5: An Ellipse? *Etc*</u>

In these video presentations, I invent some initial conditions; namely, D_p, the distance of the planet from the sun at perihelion, and V_{max}, its velocity at that point in time. Using the dV/d-theta circle,



dV / dθ circle

it is easy to show that:

$$\frac{V_{max}}{V_{min}} = \frac{1 + e}{1 - e}$$

And using the conservation of angular momentum, it follows that

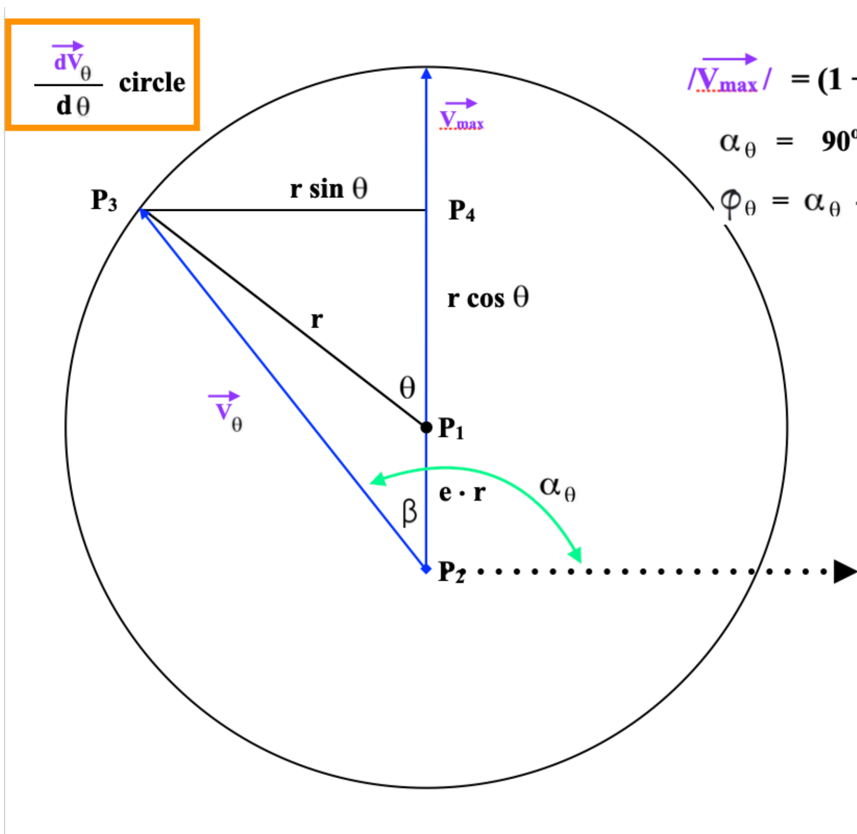
$$\frac{D_a}{D_p} = \frac{1 + e}{1 - e}$$

Where D_a is the distance of the planet from the sun at aphelion.

I go on to show that IF the orbit is an ellipse, then it follows that e, as defined above with reference to the dV/d-theta circle, must in fact be the eccentricity of that ellipse. Furthermore, IF the orbit is an ellipse, knowing $\mathcal{D}_p$ and e allows us to find a, the semi-major axis of that ellipse:

$$a = \frac{1}{1 - e} \mathcal{D}_p$$

Now, I go on to prove that the orbital path is in fact an ellipse. Take the velocity of the planet, $\mathbf{V}_{theta}$, at any arbitrary point in its orbit, and consider the following diagram that appears in the Part 5.5 video (note the different vector notation):



Using trigonometry and trigonometric identities, I go on to show that for the planet at that point with this velocity vector:

$$\sin \varphi = \frac{r\,(1 + e \cos\theta)}{v_\theta}$$

Then using formulas derived earlier, it follows that:

$$\mathcal{D}_\theta = \frac{h}{r\,(1 + e\cos\theta)}$$

and furthermore that:

$$\mathcal{D}_\theta = \frac{(1 + e)\,\mathcal{D}_p}{r\,(1 + e\cos\theta)}$$

$$= \frac{a\,(1 - e^2)}{r\,(1 + e\cos\theta)}$$

This latter derivation is the standard form for the equation of an ellipse in polar coordinates with the origin at one focus.

Part 6: Proof of Kepler's Third Law

Using several of the formulas derived earlier, it is a simple matter to prove Kepler's third law:

$$T = \frac{\textbf{Area within an elliptical orbit}}{\textbf{Rate at which area is swept out by planet}}$$

$$T = \frac{\pi ab}{dA/dt}$$

→ *substitute:* **$dA/dt = h/2$**

$$T = \frac{\pi ab}{h/2}$$

→ *square both sides*

$$T^2 = \frac{4\pi^2 a^2 b^2}{h^2}$$

→ *substitute:* **$h^2 = (1 + e)GMD_p$**
and: **$b^2 = a^2(1 - e^2)$**

$$T^2 = \frac{4\pi^2 a^4 (1 - e^2)}{(1 + e)GMD_p}$$

→ *factor the difference of two squares and substitute:* **$Dp = a\,(1 - e)$**

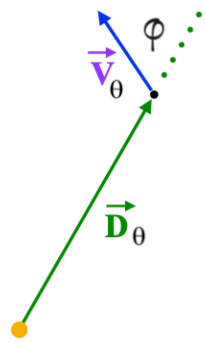
$$T^2 = \frac{4\pi^2 a^4 (1 - e)(1 + e)}{(1 + e)GMa(1 - e)}$$

→ *simplify*

$$\boxed{T^2 = \frac{4\pi^2 a^3}{GM}}$$

<u>Part 7: Final Exam</u>

You are given the speed and distance from the sun of planet Q, as well as the angle phi defined by its displacement and velocity vectors, at a particular moment in time. However, you are not given its angle of azimuth relative to the major axis.



$GM = 40\ AU^3/yr^2$

$/\vec{D}_\theta/ = 1.28$ AU

$/\vec{V}_\theta/ = 7.321$ AU/yr

$\varphi = 1.119$ radians

Can you find the equation for its orbit?